INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

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N° 5173

THÈME 1

Rapport de recherche

ISSN 0249-6399 ISRN INRIA/RR--5173--FR+ENG

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Thème 1 — Réseaux et systèmes
Projet Hipercom

Rapport de recherche n° 5173 — — 6 pages

Abstract: We provide analytical results about the performance of various Connected Dominating Set (CDS) algorithms: MultiPoint Relaying (MPR) flooding, MPR-CDS, Generalized Wu Li CDS (GWL-CDS). In particular we focuse on the 1D unit disk graph model.

Key-words: Mobile ad hoc, connected dominating set, unit disk model, asymptotic results.

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Résultats analytiques sur les ensembles dominants connectés dans les réseaux mobiles ad hoc

Résumé : Nous présentons des résultats analytiques sur les performances de différents algorithmes sur les ensemble dominants connectés (CDS): innondation par Relais MultiPoints (MPR), MPR-CDS, algorithmes Wu et Li généralisé (GWL-CDS). En particulier nous nous intéressons au modèle du graphe avec disque unité en dimension 1.

Mots-clés : Mobile ad hoc, ensemble dominants connectés, modèle du disque unité, résultats asymptotiques.

1 Introduction

In [2, 3, 1] we describe many connected dominating set built from Multipoint Relay method (MPR). In [4] there is a description of Genaralized Wu and Li algorithm for oriented graph that we simplified in [5] for the context of non oriented graphs. In this note we analyze the performance of these algorithm via analytical models. We investigate the case where the network is a unit disk graph of dimension 1. We assume that the nodes are randomly and uniformly dispatched on an infinite segment. The radio range is 1 and the density of nodes is $\nu/2$. This implies that the average node degree is ν.

In particular we will prove that the densities of MPR connecting sets tend to constant densities when $\nu \to \infty$. The density of CDS nodes of generalized Wu Li algorithm also tends to a constant when selection criterium is based on ID, but tends to be of order $\sqrt{\nu}$ when the criterium is the node degree. Therefore this confirms the poor performance of generalized Wu and Li algorithm with degree criterium.

2 MPR connected dominating sets

There are two kinds of MPR connected dominating set: the set obtained by the transmitter in an MPR flooding and the set obtained by the specific algorithm described in [3], called MPR-CDS. The former has the advantage to enable less transmitters than the latter but with the drawback that the connected dominating set is source dependent (*i.e.* the connected dominating set varied when the source of the initial broadcast varies). In fact the MPR flooding is last hop dependent.

Theorem 1 *The probability that a node belongs to the MPR flooding set tends to be equivalent to* $\frac{2}{\nu}$.

The proof is in [2, 1].

Theorem 2 *The probability that a node belongs to the MPR CDS set tends to be equivalent to* $\frac{3}{\nu}$ *when* $\nu \to \infty$.

It has been proven in [2] that a node has 2 MPR which are the extremal points of its coverage interval. Conversely the number of MPR selector neighbors is 2. A node belongs to the MPR CDS (i) if it has the shortest ID in its neighborhood, which occurs with probability $\frac{1}{\nu}$, or (ii) if one of its MPR selector has the shortest ID in the neighborhood, which occurs with probability $\frac{2}{\nu}$. These two events being exclusive the probabilities sum.

3 Generalized Wu Li algorithms

By ID criterium we assume that a node belongs to the CDS when the neighbors with higher ID don't form a connected component that covers the whole node neighborhood.

Theorem 3 *The probability that a node belongs to the generalized Wu and Li CDS with ID criterium tends to be equal to $\frac{4}{\nu}$ when $\nu \to \infty$*

In fact rule k is equivalent to rule 2 in dimension 1. It has been shown in [2] that the average number of CDS members in a segment of length x is exactly $2x - 1$.

By degree criterium we assume that a node belongs to the CDS when the neighbors with higher degree don't form a connected component that covers the whole node neighborhood.

Theorem 4 *The probability that a node belongs to the generalized Wu and Li CDS with degree criterium tends to be equal to $\sqrt{8\pi\nu}$ when $\nu \to \infty$.*

Let $N(x)$ be the number of neighbor of a node at location x on the segment map. Let $I([a,b])$ be the number of nodes contained by interval $[a,b]$. Therefore $N(x) = I([x-1, x+1])$. Let $\Delta(x) = N(x) - N(0)$. We need the following lemma

Lemma 1 *For any $x \in [-1, 0]$ and $y \in [0, 1]$ $\Delta(x)$ and $\Delta(y)$ are independent variables.*

Proof: If $x \in [0,1]$, then we have $\Delta(x) = I([-1, -1+x]) - I([1, 1+x])$. If $x \in [-1, 0]$ then $\Delta(x) = I([-1+x, -1]) - I([1+x, 1])$. Since the intervals don't overlap then $\Delta(x)$ and $\Delta(y)$ are independent when x and y have different signs.

Let R_r be the absolute value of the first $x \in [0,1]$ such that $\Delta(x) > 0$ $(\forall t \in [0,x] \Delta(x) \leq 0)$. Let R_l be the absolute value position of the first $y \in [-1, 0]$ such that $\Delta(x) > 0$ $(\forall t \in [y, 0] \Delta(x) \leq 0)$. We know that R_r and R_l are independent. A node at position 0 belongs to the CDS iff $R_r + R_l > 1$

Lemma 2 *Let $x \in [0,1]$ the probability $P(x)$ such that $R_r = x$ has Laplace transform*

$$\int_0^\infty P(x) e^{-\omega x} dx = \frac{2 + \frac{2\omega}{\nu} + \sqrt{(2 + \frac{2\omega}{\nu})^2 - 4}}{2}$$

Having $\Delta(t) \leq 0$ for all $t \in [0, x]$ is equivalent that an M/M/1 system with service rate and arrival rate equal to $\frac{\nu}{2}$ starts with one customer and does not empty its queue during a time interval of length x. Let $f(\omega)$ be the Laplace transform of the distribution of the time T needed to empty the queue $f(\omega) = E[e^{-\omega T}]$. Let θ be the time needed for the exit of the first customer, we have from classic queueing theory:

$$T = \theta + N_\theta \times T \tag{1}$$

where N_θ is a Poisson random variable of mean θ and $N \times T$ means the addition of N independent copies of T (N i.i.d. variables distributed as T). Therefore

$$\begin{aligned} f(\omega) &= \int_0^\infty P(\theta = x) e^{-x\omega} e^{x\frac{\nu}{2}(f(\omega)-1)} dx && (2) \\ &= \frac{\nu/2}{\nu/2(1 - f(\omega) + \omega} && (3) \end{aligned}$$

Consequently the Laplace transform of the distribution of $R_r + R_l$ is $f(\omega)^2$. Therefore the probability $P(\nu)$ that node at position 0 belongs to the CDS, that is $R_l + R_r > 1$, satisfies

$$P(\nu) = \frac{1}{2i\pi}\int_{-i\infty}^{i\infty} \frac{1-(\omega+1+\sqrt{(\omega+1)^2-1})^2}{w} e^{\nu\omega} d\omega$$

Using the fact that $1-(\omega+1+\sqrt{(\omega+1)^2-1})^2 \sim 2\sqrt{\omega} + O(\omega)$ when $\omega \to 0$ we have from Flajolet and Odlyzko theorem:

$$\frac{1}{2i\pi}\int_{-i\infty}^{i\infty} \frac{1-(\omega+1+\sqrt{(\omega+1)^2-1})^2}{w} e^{\nu\omega} d\omega \sim \frac{2\Gamma(1/2)}{\pi}\nu^{1/2}.$$

We display in figure 1 the various node densities of the connected dominating set studied in this note.

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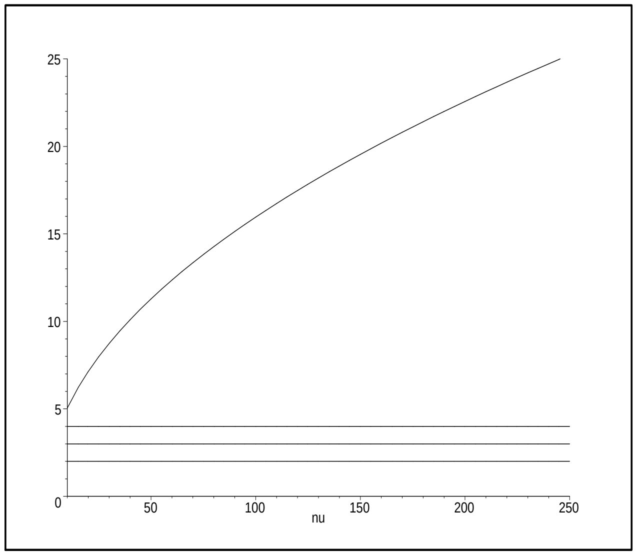

Figure 1: Node density of various CDS algorithm from bottom to top: MPR flooding, MPR-CDS, GWL-CDS with ID, GWL-CDS with degree.

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http://www.inria.fr
ISSN 0249-6399